GCSE Maths – Algebra

Forming and Solving Equations

Notes

WORKSHEET

Forming and Solving Equations

To find an unknown value in a real-life scenario, we can use algebra to help us to solve the problem using the other information we have.

The first step to approaching these problems is to always **identify the unknown values as letters (algebra)**, such as x or y. We can then **create an equation** using the operations we are aware of (e.g. multiplication, squared, addition), and proceed to **solve the equation** in the normal fashion.

For revision on solving equations see notes:

- 'Algebra – Solving Linear Equations'
- 'Algebra – Solving Quadratic Equations'

Example: Clara, Marty and Karen each win a share of a raffle prize. Clara wins twice as much as Marty, and Karen wins £50 less than Clara.
If the total prize money was £400, how much money did Marty win?

1. **Identify the unknown value** which we are trying to find.

   We are looking for the amount of money Marty won.

   $$Let\ the\ amount\ of\ money\ won\ by\ Marty\ =\ x$$

2. Use the information in the question to express Clara and Karen's winning **in the form of** x.

   Clara wins twice as much as Marty: *Clara wins* £$\mathbf{2x}$.

   Karen wins £40 less than Clara: *Karen wins* £$(\mathbf{2x - 50})$.

3. **Create an equation** using the amount won by each winner and other information we are given.

   The total winnings is £400, *so the sum of Marty, Clara and Karen's winnings must be* £400.
   $$x\ +\ 2x\ +\ (2x - 50)\ =\ 400$$

4. **Solve** the linear equation to find x.

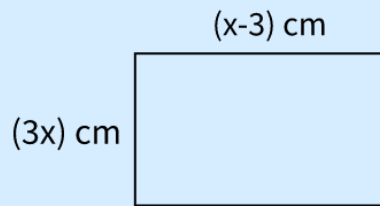 $$x\ +\ 2x\ +\ (2x - 50)\ =\ 400$$
   $$5x - 50 =\ 400$$
   $$5x = 450$$
   $$x = \frac{450}{5} = 90$$

   Marty won £90.

Example: The area of the rectangle given below is 54 cm^2. Find the perimeter.

(x-3) cm

(3x) cm

1. **Form an equation** using the area of the shape.

   We already have an unknown in this question but must find this to calculate the perimeter.
   $$3x(x - 3) = 54$$
   $$3x^2 - 9x = 54$$

2. **Solve this quadratic** equation to find x.

   $$3x^2 - 9x = 54$$
   $$3x^2 - 9x - 54 = 0$$
   $$x^2 - 3x - 18 = 0$$
   $$(x + 3)(x - 6) = 0$$
   $$x = 3 - \text{ or } x = 6$$

   Since length cannot be negative, x *must be a positive number, so $x = 6$.*

3. **Form an equation** for the **perimeter** of the rectangle.

   $$P = 2(3x) + 2(x - 3)$$
   $$P = 6x + 2x - 6$$
   $$P = 8x - 6$$

5. **Substitute** the value for x into the formula for the perimeter.

   $$P = 8x - 6$$
   $$P = 8(6) - 6$$
   $$P = 48 - 6$$
   $$P = 42$$

The perimeter of the shape is **42 cm.**

If we are looking to find **two unknown values**, we have to form **two simultaneous equations.** For revision on simultaneous equations see notes 'Algebra – Simultaneous Equations'.

Example: Marge thinks of two positive integers. One number is four times the other, and the product of these is 64. Use this information to find the numbers which Marge was thinking of.

1. **Identify the unknown values** we are trying to find as **letters**.

$$Let\ the\ smaller\ integer\ =\ a$$

$$Let\ the\ bigger\ integer\ =\ b$$

2. **Express the relationships** between a and b as equations.

We know one number is four times the other:

$$4a = b \quad \textit{(Equation 1)}$$

We know the product of the numbers is 64:

$$ab = 64 \textit{ (Equation 2)}$$

3. **Solve** the simultaneous equations.

Substitute Equation 1 into Equation 2 to find a:

$$ab = 64$$
$$a(4a) = 64$$
$$4a^2 = 64$$
$$a^2 = 16$$
$$a = \pm4$$

We know the values are positive integers, so $\boldsymbol{a = 4}$.

Substitute $a = 4$ into Equation 1:

$$4a = b$$
$$4(4) = b$$
$$\boldsymbol{b = 16}$$

*Marge thought of the numbers **4** and **16**.*

Forming and Solving Equations – Practice Questions

1. Rose, David and Rani work for an advertising company, and are tasked with selling festival tickets. Rose sells 100 more than David and Rani sells three times as much as David. All together they sell 800 tickets. How many tickets did David sell?

2. Michelle is saving up for a handbag costing £108. She works for one week at £6.50 per hour, and at the end of the week, her grandmother gives her £17. If by this time she has the exact amount to purchase the handbag, how many hours did she work that week?

3. Calculate the side length of the square below:

$15y$

$4y^2-4$
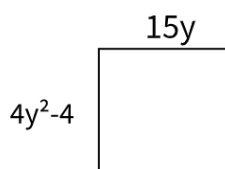
4. Martin and Jason have a 39-year age gap between them. If Marten is four times Jason's age, how old is Jason?

5. Polly thinks of a number. She doubles the number and adds six.

   Queenie thinks of a different number. She multiplies the number by eight and subtracts six.

   Polly and Queenie both end on the same number. The number Polly starts with is twice Queenie's.

   Use the information to find what number Polly and Queenie started with.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.